Newbold School Calculation Policy

At Newbold School, we strive to help our children understand the importance and relevance of Maths in our lives. We do this with hands-on activities and investigations and by ensuring that children see that maths is important across the whole curriculum. We want our children to become confident mathematicians.

Newbold School follows the New Curriculum 2014. The emphasis is on promoting fluency, reasoning and problem solving. It aims to make learning consistent across the school in order to ensure pupils grip the basic calculations.

It can be very confusing for a child to use one method of calculation at home and a different one in school. It may be helpful for parents/carers to look through our Calculation Policy to be aware of how Newbold School will approach calculations in class. All staff at Newbold School will be happy to fully explain or clarify a calculation method to ensure a smooth home/school link.

The Early Years Goals, set out the goals for children from Foundation 1 and Foundation 2.

The following links will take you to the relevant section of this document where the key objectives in each age related stage are set out.

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Maths in the Early Years

Maths is a specific area of learning and is split into 2 areas: 'Numbers' and 'Shape, Space and Measure'. The developmental statements are within age/stage bands but we emphasize that every child develops at their own rates and in their own ways.

In F1 (age 3-4 years) we tend to see most development within the 30-50 month age band and some aspects of the 40-60+ month age band. The goals for the end of Early Years (end of F2) is to achieve all the statements in 40-60+ and the overall Early Learning Goal.

NUMBERS:

30-50 months:

- Uses some number names and number language spontaneously.
- Uses some number names accurately in play.
- Recites numbers in order to 10.
- Knows that numbers identify how many objects are in a set.
- Beginning to represent numbers using fingers, marks on paper or pictures.
- Sometimes matches numeral and quantity correctly.
- Shows curiosity about numbers by offering comments or asking questions.
- Compares two groups of objects, saying when they have the same number.
- Shows an interest in number problems.
- Separates a group of three or four objects in different ways, beginning to recognise that the total is still the same.
- Shows an interest in numerals in the environment.
- Shows an interest in representing numbers.
- Realises not only objects, but anything can be counted, including steps, claps or jumps.

40-60+ months:

Recognise some numerals of personal significance.

- Recognises numerals 1 to 5.
- Counts up to three or four objects by saying one number name for each item.
- Counts actions or objects which cannot be moved.
- Counts objects to 10, and beginning to count beyond 10.
- Counts out up to six objects from a larger group.
- Selects the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Counts an irregular arrangement of up to ten objects.
- Estimates how many objects they can see and checks by counting them.
- Uses the language of ‘more’ and ‘fewer’ to compare two sets of objects.
- Finds the total number of items in two groups by counting all of them.
- Says the number that is one more than a given number.
- Finds one more or one less from a group of up to five objects, then ten objects.
- In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting.
- Records, using marks that they can interpret and explain.
- Begins to identify own mathematical problems based on own interests and fascinations.

EARLY LEARNING GOAL:

Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.

Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

They solve problems, including doubling, halving and sharing.

SHAPE, SPACE AND MEASURE:

30-50 months:

- Shows an interest in shape and space by playing with shapes or making arrangements with objects.
- Shows awareness of similarities of shapes in the environment.
- Uses positional language.
- Shows interest in shape by sustained construction activity or by talking about shapes or arrangements.
- Shows interest in shapes in the environment.
- Uses shapes appropriately for tasks.
- Beginning to talk about the shapes of everyday objects, e.g. 'round' and 'tall'.

40-60 months:

Beginning to use mathematical names for 'solid' 3D shapes and 'flat' 2D shapes, and mathematical terms to describe shapes.

- Selects a particular named shape.
- Can describe their relative position such as 'behind' or 'next to'.
- Orders two or three items by length or height.
- Orders two items by weight or capacity.
- Uses familiar objects and common shapes to create and recreate patterns and build models.
- Uses everyday language related to time.
- Beginning to use everyday language related to money.
- Orders and sequences familiar events.
- Measures short periods of time in simple ways.

EARLY LEARNING GOAL

Children use everyday language to talk about size, weight, capacity, position, distance, time and money to compare quantities and objects and to solve problems.

They recognise, create and describe patterns. They explore characteristics of everyday objects and shapes and use mathematical language to describe them.

Maths in Year 1

Working mathematically

By the end of year 1, children begin to solve simple problems involving addition and subtraction in familiar contexts such as going shopping, using a range of hands-on equipment, symbols, images and pictures. They begin to use what they know to tackle problems that are more complex and provide simple reasons for their opinions.

Number

- **Counting and understanding numbers**

Children will identify and represent numbers using objects, pictures and models, such as the number line, and use 'equal to, more than, less than (fewer), most and least.' Children will accurately count numbers to, and across, 100 forwards and backwards from any given number with increasing understanding. They count, read, write and order numbers in numerals up to 100 and from 1 to 20 in words. When given a number, they can identify one more and one less. They can count in multiples of twos, fives and tens.

- **Calculating**

Children will understand known addition and subtraction facts within 20, including zero. They will demonstrate an understanding of multiplication and division through grouping and sharing using hands-on resources, pictorial representations and arrays (2, 5 and 10). They understand doubling and halving small quantities**.**

- **Fractions**

Through play and hands-on resources, children will find and name half and one quarter of objects, shapes and quantities.

Measurement

Children will begin to measure using non-standard units (finger widths, blocks etc.) moving to standard units of measure (e.g. cm) using tools such as a ruler, weighing scales and containers. They will begin to record and compare measurements such as lengths and heights, mass and weight, capacity and volume using language such as long / short; heavy / light; full / half-full / empty. They will tell the time to the hour, half past the hour and be able to sequence events in chronological order using precise language (for example, before and after, next, first, today etc.). Children will recognise and know the value of different denominations of coins and notes.

Geometry

Children will recognise and name common 2-D shapes, e.g. rectangles (including squares), circles and triangles, and 3-D shapes, e.g. cuboids (including cubes, pyramids and spheres) in different orientations and sizes. They will describe position, direction and movement, including whole, half and three quarter turns.

Statistics

In preparation for year 2, children will begin to compare, sort and classify information, including through cross curricular links e.g. science – sorting materials into groups according to their properties. They will also begin to construct simple pictograms and tables.

Maths in Year 2

Working mathematically

By the end of year 2, children will solve problems with one or a small number of simple steps. Children will discuss their understanding and begin to explain their thinking using appropriate mathematical vocabulary, hands-on resources and different ways of recording. They will ask simple questions relevant to the problem and begin to suggest ways of solving them.

Number

- **Counting and understanding numbers**

Children will develop their understanding of place value of numbers to at least 100 and apply this when ordering, comparing, estimating and rounding. Children begin to understand zero as a place holder as this is the foundation for manipulating larger numbers in subsequent years. Children will count fluently forwards and backwards up to and beyond 100 in multiples of 2, 3, 5 and 10 from any number. They will use hands-on resources to help them understand and apply their knowledge of place value in two digit numbers, representing the numbers in a variety of different ways.

- **Calculating**

Children learn that addition and multiplication number sentences can be re-ordered and the answer remains the same (commutativity) such as 9+5+1= 5+1+9. They learn that this is not the case with subtraction and division. They solve a variety of problems using mental and written calculations for +, -, x, ÷ in practical contexts. These methods will include partitioning which is where the number is broken up into more manageable parts (e.g. 64 = 60 + 4 or 50 + 14), re-ordering (e.g. moving the larger number to the beginning of the number sentence when adding several small numbers) and using a number line. Children will know the 2, 5 and 10 times tables, as well as the matching division facts (4 x 5 = 20, 20 ÷ 5 = 4) and can recall them quickly and accurately. They apply their knowledge of addition and subtraction facts to 20 and can use these to work out facts up to 100.

- **Fractions including decimals**

Throughout year 2, children will develop their understanding of fractions and the link to division. They explore this concept using pictures, images and hands-on resources. They will solve problems involving fractions (e.g. find 1/3 of the hexagon or ¼ of the marbles) and record what they have done. They will count regularly and fluently in fractions such as ½ and ¼ forwards and backwards and, through positioning them on a number line, understand that some have the same value (equivalent) e.g. ½ = ¼.

Measurement

Children will estimate, choose, use and compare a variety of measurements for length, mass, temperature, capacity, time and money. By the end of year 2, they will use measuring apparatus such as rulers accurately. They will use their knowledge of measurement to solve problems (e.g. how many ways to make 50p). They extend their understanding of time to tell and write it on an analogue clock to 5 minute intervals, including quarter past / to the hour. They will know key time related facts (minutes in an hour, hours in a day) and relate this to their everyday life.

Geometry

Children will identify, describe, compare and sort common 2-D and 3-D shapes according to their properties (sides, vertices, edges, faces) and apply this knowledge to solve simple problems. They develop their understanding by finding examples of 3-D shapes in the real world and exploring the 2-D shapes that can be found on them (e.g. a circle is one of the faces on a cylinder). Children begin to describe position, direction and movement in a range of different situations, including understanding rotation (turning through right angles clockwise and anti-clockwise). They use their knowledge of shape in patterns and sequences.

Statistics

Children sort and compare information, communicating findings by asking and answering questions. They will draw simple pictograms, tally charts and tables.

Maths in Year 3

Working mathematically

By the end of year 3, children will talk about their mathematics using the numbers they are familiar with, applying their understanding of number, measures and shape to a greater range of problems. They will make decisions about calculations and information that is needed to solve problems, for example when a recipe for two people needs to be doubled to make a recipe for four. Children will be expected to prove their thinking through pictures, jottings and conversations. They will be encouraged to pose their own questions, working in an organised way to solve them which will help pupils to identify common patterns or any errors more easily.

Number

- **Counting and understanding numbers**

Children will be very familiar with numbers that have 3 digits and will have experienced many opportunities to order, compare and show them in different ways using apparatus such as a tape measure, a 100 grid or money. Using their understanding of place value (how the value of each digit changes depending on its position in the number), children will be able to partition (break and make) numbers in different ways e.g. 234 = 200 and 30 and 4; 100 and 100 and 20 and 10 and 4; or 200 and 20 and 14. They will develop a secure understanding of numbers up to 1000 and will count beyond it in 1s, 10s and 100s. They will use this counting to help find 10 or 100 more than any given number.
Children will be introduced to numbers with one decimal place and will count up and down in tenths; share groups of objects or shapes into tenths and represent these in pictures and using hands-on resources.
Children will count forwards and backwards from 0 in steps of 4, 8, 50 and 100 and link this to multiplication and division. They will also count in 3s to help maintain their fluency from Year 2.

- **Calculating**

Children will continue to develop their mental calculation skills to add and subtract combinations of three-digit numbers e.g. 248 +/- 8; 319 +/- 40; 428 +/- 200. They will develop their range of strategies using jottings (sketches and notes to help them remember the steps) and number lines to help them understand how each calculation works. Children will share their methods with others to help them see which work best, are quickest and most accurate. Children will understand the importance of estimation when calculating to see if their answer is reasonable or not. They will recall their multiplication and division facts for 3, 4 and 8x tables and be supported to see the links between the 2, 4 and 8x tables. They explore patterns and rules for the times tables they learn and will use pictures and objects to support their understanding. They will also learn that multiplication can be done in any order e.g. 3 x 4 x 2 = 2 x 3 x 4.

Children will be introduced to more formal methods of recording addition and subtraction, including column methods. They will use hands-on resources to secure their understanding of these methods. This will be applied to numbers up to three digits. Children who become very adept at these calculations will be stretched through problems such as those involving missing numbers so that they know when, if and why they need to use these methods.

Children will develop their understanding of multiplication and division and apply their times table knowledge to multiply 2-digit by 1-digit numbers using the skills of partitioning (breaking and making numbers). For example, 43 x 5 can also be thought of as 40 x 5 and 3 x 5 or (4 x 5 x 10) + (3 x 5). They will move from informal methods of calculating multiplication and division to formal written methods i.e. short column multiplication and be supported by using hands-on resources.

- **Fractions**

Children will develop their understanding of fractions and decimals and will be introduced to tenths. They will count and understand tenths as ten equal parts as well as through dividing sets of objects into ten equal parts / groups. They will find and write fractions of objects using their multiplication tables knowledge, e.g. 1/5 of a group of 20 buttons can be solved by 20 ÷ 5 = 4, and will continue to explore equivalent fractions using diagrams to explain their understanding e.g. 2/4 is equivalent to or of equal value to 4/8. They will also begin to add and subtract fractions where the denominator is the same e.g. 4/6 + 1/6 = 5/6.

Measurement

Children will continue to measure, compare, add and subtract measurements and progress to mixed units e.g. expressing amounts as litres and millilitres – 2 litres 400ml. They will measure the perimeter of 2-D shapes and will continue to add and subtract amounts of money including giving change. Children will estimate and read time to the nearest minute on analogue and digital clock faces. They will be introduced to the Roman numerals I to XII to help with this. Problem solving and calculating with time will involve comparing the duration of events such as the length of favourite television programme or journeys to school. They will use language with increasing accuracy, such as seconds, minutes and hours; o'clock, a.m. / p.m., morning, afternoon, noon and midnight. They will need to recall the number of seconds in a minute and the number of days in each month, year and leap year.

Geometry

Children will accurately draw 2-D shapes with rulers measuring sides accurately. They will make 3-D shapes to help them understand how they are composed and will recognise 3-D shapes in a range of places and contexts (e.g. buildings, packages) and use correct mathematical vocabulary to describe them. They will learn what a right angle is and know that two right angles make a half-turn, three make three quarters of a turn and four a complete turn as well as identify whether angles are greater than or less than a right angle . They will also be able to identify horizontal and vertical lines and pairs of perpendicular (⊾) and parallel lines (=).

Statistics

Children will collect, organise, answer and pose questions about information using bar charts, pictograms and tables to answer questions such as ‘how many more children prefer football to cricket?’.

Maths in Year 4

Working mathematically

By the end of year 4, children will apply their understanding of maths to solve a wide variety of problems with more than one step and be expected to prove their thinking through pictures, jottings and conversations. They will continue to make connections between different areas of maths and ask their own questions, working in an organised way to find solutions which help them identify common patterns or any errors more easily.

Number

- **Counting and understanding numbers**

Children will be very familiar with numbers that have up to 4 digits and will be able to order and compare by showing them in different ways such as on a tape measure or using handson resources. Using their understanding of place value (how the value of each digit changes depending on its position in the number), children will be able to partition (break and make) numbers in different ways e.g. 2345 = 2000 and 300 and 40 and 5 but could also represent this as 1000 and 1000 and 200 and 100 and 40 and 5 or 2000 and 200 and 145. They will work with numbers securely up to 10,000 and may begin to count beyond in 1s, 10s, 100s and 1000s. They will use this to help them find 10, 100 or 1000 more or less than any given number. They will multiply and divide whole numbers by 10 and 100 and understand that this changes the value of each digit rather than 'just adding a 0'. They will develop their understanding to decimal hundredths, comparing and ordering these using contexts such as money. Children will also learn about the pattern to find any Roman numeral to 100. Children will develop their expertise when counting forwards and backwards from 0 to include multiples of 6, 7, 9 and 25; decimals with up to 2 places and fractions. They will be able to fluently count in tenths, hundredths and simple fractions. They will develop their understanding of negative numbers through counting backwards through 0. Children will be able to recognise and describe number patterns and relationships including multiples (e.g. 3, 6, 9, 12 are multiples of 3) and factor pairs (e.g. 1 and 12, 2 and 6, 3 and 4 are all factor pairs for 12) for known times tables.

- **Calculating**

Children will develop various strategies for solving +, -, x, ÷ calculations mentally, using jottings when appropriate and for checking that their answers are sensible. Children will be encouraged to share their methods with others to help them see which work best, are quickest and most accurate. Over the course of the year, children will become fluent in all multiplication and division facts up to 12 x 12 and apply these facts to other problems e.g. 232 x 7 = (200 x 7) + (30 x 7) + (2 x 7). Children will use the = sign to demonstrate equal value e.g. 3 x 8 = 48 ÷ 2 and solve missing number problems e.g. 3 x ? = 48÷2. They will explore patterns and rules for the times tables they learn and use pictures and objects to support their understanding.
Children will be required to solve problems accurately using the column addition and subtraction methods for numbers with up to 4-digits and explain how the methods work.

They will use apparatus to secure their understanding of these. This will include addition and subtraction calculations with different numbers of digits (such as 1286 + 357); and numbers containing 0s (such as 8009 – 3231). They will use formal written methods of short multiplication and short division for two and three digit numbers by a single digit. Children who become very adept at these types of calculations will be stretched through problems such as those containing missing numbers so that they know when, if and why they need to use the methods.

- **Fractions including decimals**

Children will develop their understanding of fractions by comparing to, or finding a part of, the whole. Through hands-on resources, pictures or jottings, such as a number line, children will add and subtract two fractions with the same denominator (e.g. 2/3 + 2/3). Children will solve problems involving fractions such as 'find ¾ of 20 litres' using their knowledge of multiplication and division and through practical equipment. Children secure their understanding that fractions and decimals are different ways of expressing numbers and proportions.

Measurement

Children secure their understanding of place value and decimals to record measurements accurately. They use their understanding of multiplying and dividing by 10, 100 and 1000 to convert between different units of measure of length (km, m, cm, mm), weight (kg, g) and money (£ and p). Children will link their understanding of area to multiplication and describe how to find the perimeter of a rectangle quickly. Children will read and write the time accurately using analogue and digital clocks, including clocks with Roman numerals. They will convert between units of time (hours, minutes and seconds). Children estimate, compare, calculate and solve a variety of problems involving all units of measurement.

Geometry

Children will extend their knowledge of shape to include more unusual quadrilaterals (foursided shapes) and triangles. They will use increasingly more specific vocabulary such as parallelogram, rhombus and trapezium; scalene and isosceles. They refine their understanding of symmetry and solve problems where the shape is not displayed in its usual way (e.g. it might be on its side). Children find and name different angles and use this information to decide if a shape is regular or irregular. Children describe position and movement on a grid as co-ordinates and will plot points to draw 2-D shapes.

Statistics

Children will complete, read and interpret information on bar charts; they will solve problems that involve finding information in charts, tables and graphs; including time graphs.

Maths in Year 5

Working mathematically

By the end of year 5, children will apply their mathematical experiences to explore ideas and raise relevant questions, constructing complex explanations and reasoned arguments. They will be able to solve a wide variety of complex problems which require sustained concentration and demand efficient written and mental methods of calculations. These will include problems relating to fractions, scaling (times as many), converting between units of measure and employ all four operations (+, -, x, ÷).

Number

- **Counting and understanding numbers**

Children extend and apply their knowledge of place value for numbers up to one million, rounding, estimating and comparing them (including decimals and negative numbers) in a variety of situations. They are introduced to powers of ten and are able to count forwards or backwards from any number (for example, -50, -5… 5, 50, 500, 5000...). Through investigations, they will discover special numbers including factors, primes, square and cube numbers.

- **Calculating**

Children will be fluent in a wide range of mental calculation strategies for all operations and will select the most appropriate method dependent on the calculation. They apply their knowledge of place value fluently to multiply and divide numbers (including decimals) by 10, 100 and 1000. When mental methods are not appropriate, they use formal written methods of addition and subtraction accurately. They continue to develop their understanding of the formal methods through hands-on resources and use their known facts within long multiplication (up to 4 digit numbers by 2 digit numbers e.g. 2345 x 68) and short division (up to 4 digit numbers by 1 digit number e.g. 2345 ÷ 7) which may result in remainders. They solve multi-step problems in meaningful contexts and decide which operations to use.

- **Fractions including decimals and percentages**

Children secure their strong understanding that fractions express a proportion of amounts and quantities (such as measurements), shapes and other visual representations. Children extend their knowledge and understanding of the connections between fractions and decimals to also include percentages. They will be able to derive simple equivalences (e.g. 67% = 67/100 = 0.67) and recall percentage and decimal equivalents for ½, ¼, 1/5, 2/5, 4/5 and fractions with a denominator of a multiple of 10 or 25 (e.g. 25% = 25/100). They order, add and subtract fractions, including mixed numbers and those whose denominators are multiples of the same number, for example + = + = = . Using apparatus, images and models, they multiply proper fractions and mixed numbers by whole numbers. Children continue to develop their understanding of fractions as numbers, measures and operators by finding fractions of numbers and quantities in real life situations.

Measurement

Through a wide variety of practical experiences and hands-on resources, children extend their understanding of measurement. They convert larger to smaller related units of measure and vice-versa including length, capacity, weight, time and money. Children will convert between imperial (such as inches, pints, miles) and metric units (such as centimetres, litres, kilometres). Children will measure, calculate and solve problems involving perimeter of straight-sided, right-angled shapes (rectilinear) and learn to express this algebraically such as, $4 + 2b = 20$. They find and measure the area of these shapes with increasing accuracy. They begin to estimate volume.

Geometry

Children will measure, identify and draw angles in degrees, developing a strong understanding of acute, obtuse, reflex and right angles. They use this knowledge to find missing angles and lengths in a variety of situations, including at a point, on a straight line and within a shape. Children will move (translate), reflect shapes and describe their new positions. Language will be used with increasing sophistication to compare and classify shapes based on their properties and size. They will be able to visualise 3-D shapes from 2-D diagrams. They will use their understanding or shapes to solve problems.

Statistics

Children will complete, read and solve comparison, sum and difference problems using information presented in graphs, charts and tables, including timetables. They begin to decide which representations of data are the most appropriate and are able to justify their reasons.

Maths in Year 6

Working mathematically

By the end of year 6, children will structure their own investigations and solve a wide variety of increasingly complex problems. They will independently develop their own lines of enquiry and be expected to prove their solutions in a variety of ways including algebra, negative proof (use a counter example to prove the rule) and be able to communicate their results using accurate mathematical language. Children will demonstrate secure knowledge and confidence to talk in depth about mathematical concepts and explain their solutions, decisions and reasoning.

Number

- **Counting and understanding numbers**

Children extend and apply their knowledge of place value for numbers up to and beyond one million (including decimals and negative numbers) in a variety of situations. Special numbers are extended to include common factors, common multiples and a deeper understanding of prime numbers. Children will be able to round numbers and identify what degree of accuracy is appropriate.

- **Calculating**

Children will be fluent in a wide range of mental and formal written calculation strategies for all operations, extending to long division (four digit numbers by two digit numbers) by the end of the year. They will apply estimation in a range of ways. Through investigations, they explore the effect of the order of operations including the use of brackets.

- **Fractions including decimals and percentages**

Children recall and using equivalences between simple fractions, decimals and percentages. Additionally, they are able to express fractions in their simplest form and calculate the decimal equivalent, for example = 3 ÷ 8 = 0.375.
Applying this understanding of equivalent fractions, children will order, add and subtract fractions (including mixed numbers and those with different denominators) by the end of the year e.g. + + □ = 1. Using hands-on resources and images, they will multiply and divide proper fractions and mixed numbers by whole numbers e.g. x = and ÷ 2 = . Children will solve problems involving the calculation of percentages linked to real life situations.

Ratio and proportion

Pupils explore ratio and proportion through real life experiences such as changing the quantities in recipes (scaling), scale drawings and maps.

Algebra

Throughout their primary experience children will have encountered algebra in a number of different situations which is drawn together and formalised in year 6. By the end of the year, they will confidently use symbols and letters to represent variables and unknowns in mathematical situations that they already understand, for example, simple formula and equivalent expressions a+b = b+a. Children will describe number sequences and missing number calculations. Measurement

Through investigation and problem solving, children convert between a range of measurement units (including both imperial and metric). Calculation of perimeter and area is extended to include parallelograms and triangles. Additionally, they will explore the relationship between area and perimeter. They will know how to calculate, estimate and compare volume of cubes and cuboids identifying when it is appropriate to use formula.

Geometry

Children will draw 2-D and build 3-D shapes with accuracy using given dimensions and angles. They will create nets of common 3-D shapes. They will consolidate their knowledge of angles within shapes and extend it to find missing angles in triangles, quadrilaterals and regular polygons. Children name parts of circles, including radius, diameter and circumference, and explore the relationships between these elements. Children will use four quadrant co-ordinate grids to describe positions, draw and translate simple shapes. Using their knowledge of the properties of shape, they will be able to predict missing co-ordinates and express these algebraically.

Statistics

Children will increase their knowledge of different data representations to include interpreting and constructing pie charts (using their knowledge of angles, fractions and percentages) and line graphs (e.g. miles to km conversion). They will know when it is appropriate to use the mean as an average and how to calculate it.

Calculations Policy

Rationale

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the new National Curriculum commencing September 2014. Through the policy, we aim to link key manipulatives (counters, helpful materials and equipment etc.) and representations in order that the children can be vertically accelerated through each strand of calculation. We know that school wide policies, such as this, can ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, it is expected that the majority of children in each class will be working at age-appropriate levels as set out in the National Curriculum 2014 and in line with school policy.

The importance of mental mathematics

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

To add and subtract successfully, children should be able to:

- recall all addition pairs to 9 + 9 and number bonds to 10
- recognise addition and subtraction as inverse operations
- add mentally a series of one digit numbers (e.g. 5 + 8 + 4)
- add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 — 70)
- partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways

(e.g. partition 74 into 70 + 4 or 60 + 14)

- use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:

- add and subtract accurately and efficiently
- recall multiplication facts to 12 x 12 = 144 and division facts to 144 ÷ 12 = 12
- use multiplication and division facts to estimate how many times one number divides into another etc.
- know the outcome of multiplying by 0 and by 1 and of dividing by 1
- understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
- recognise factor pairs of numbers (e.g. that 15 = 3 x 5, or that 40 = 10 x 4) and increasingly able to recognise common factors
- derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000)

- notice and recall with increasing fluency inverse facts
- partition numbers into 100s, 10s and 1s or multiple groupings
- understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
- understand the effects of scaling by whole numbers and decimal numbers or fractions
- understand correspondence where n objects are related to m objects
- investigate and learn rules for divisibility

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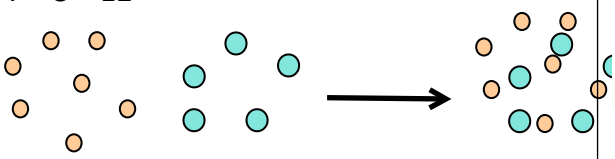
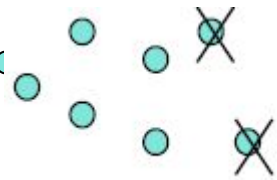
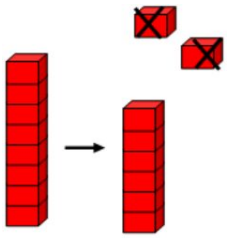
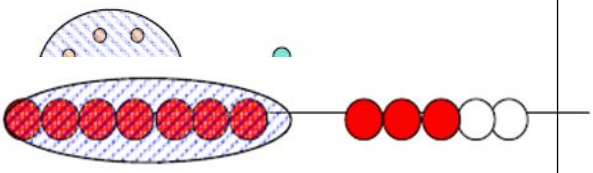
Progression in addition and subtraction

Addition and subtraction are connected.

Part	Part
Whole	

Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

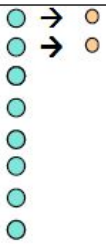
Addition	Subtraction
Combining two sets (aggregation) Putting together – two or more amounts or numbers are put together to make a total 7 + 5 = 12  Count one set, then the other set. Combine the sets and count again. Starting at 1. Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1. 	**Taking away (separation model)** Where one quantity is taken away from another to calculate what is left. 7 – 2 = 5  Multilink towers - to physically take away objects. 
Combining two sets (augmentation) *This stage is essential in starting children to calculate rather than counting* Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number. Counters:	**Finding the difference (comparison model)** Two quantities are compared to find the difference. 8 – 2 = 6 Counters:



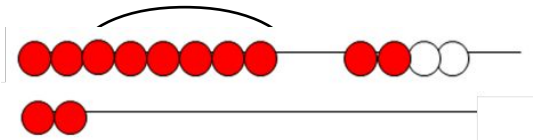
Start with 7, then count on 8, 9, 10, 11, 12

Bead strings:

Make a set of 7 and a set of 5. Then count on from 7.



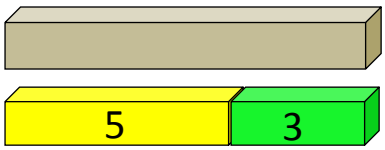
Bead strings:



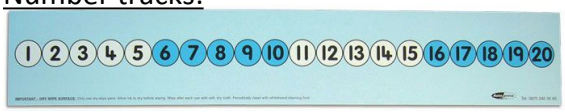
Make a set of 8 and a set of 2. Then count the gap.

Multilink Towers:

Cuisenaire Rods:

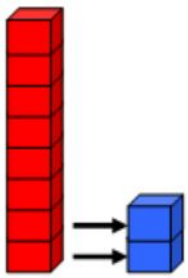


Number tracks:

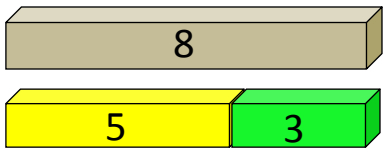


Start on 5 then count on 3 more

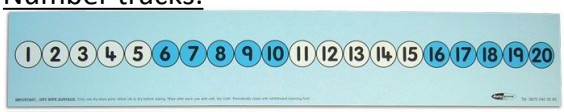
Multilink Towers:



Cuisenaire Rods:



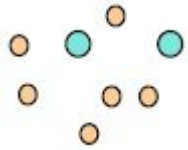
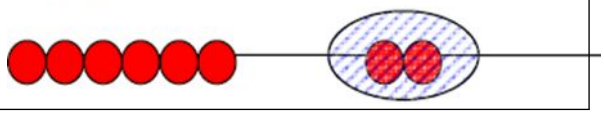
Number tracks:

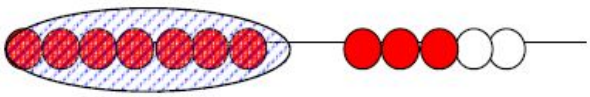
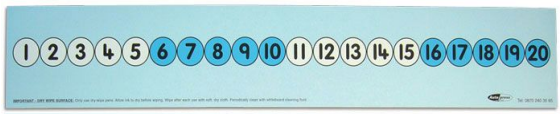
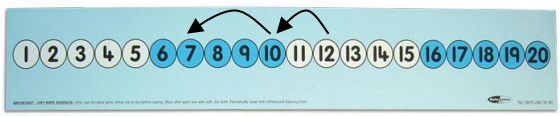
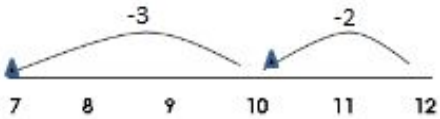


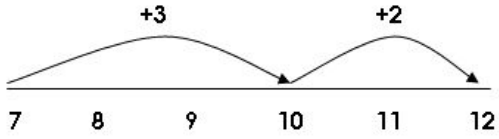
Start with the smaller number and count the gap to the larger number.

1 set within another (part-whole model)

The quantity in the whole set and one part are known, and may be used to find out how many are in the unknown part.

	8 – 2 = 6 Counters:  Bead strings: 8 – 2 = 6 

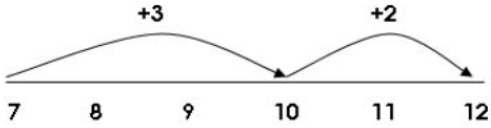
Bridging through 10s *This stage encourages children to become more efficient and begin to employ known facts.*	
Bead string:  7 + 5 is decomposed / partitioned into 7 + 3 + 2. The bead string illustrates 'how many more to the next multiple of 10?' (children should identify how their number bonds are being applied) and then 'if we have used 3 of the 5 to get to 10, how many more do we need to add on? (ability to decompose/partition all numbers applied) Number track:  Steps can be recorded on a number track alongside the bead string, prior to transition to number line. Number line	Bead string:  12 – 7 is decomposed / partitioned in 12 – 2 – 5. The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back? (ability to decompose/partition all numbers applied) Number Track:  Steps can be recorded on a number track alongside the bead string, prior to transition to number line. Number Line:  **Counting up or 'Shop keepers' method** Bead string:



12 – 7 becomes 7 + 3 + 2.
Starting from 7 on the bead string 'how many more to the next multiple of 10?' (children should recognise how their number bonds are being applied), 'how many more to get to 12?'.

Number Track:

Number Line:



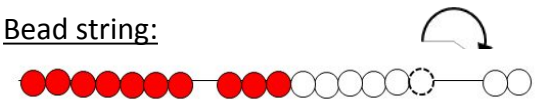
-

Compensation model (adding 9 and 11) (optional)

This model of calculation encourages efficiency and application of known facts (how to add ten)

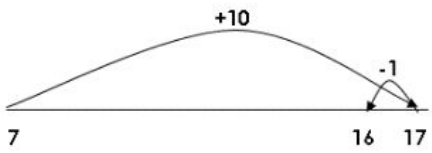
7 + 9

Bead string:



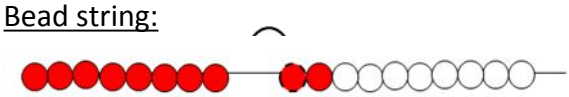
Children find 7, then add on 10 and then adjust by removing 1.

Number line:



18 – 9

Bead string:



Children find 18, then subtract 10 and then adjust by adding 1.

Number line:

Working with larger numbers

Tens and ones + tens and ones

Ensure that the children have been transitioned onto Base 10 equipment and understand the abstract nature of the single 'tens' sticks and 'hundreds' blocks

Partitioning (Aggregation model)

34 + 23 = 57

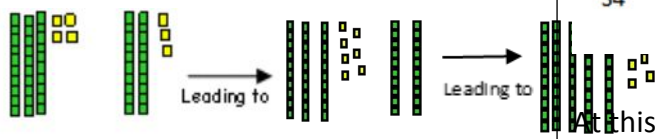
Base 10 equipment:

Children create the two sets with Base 10 equipment and then combine; ones with ones, tens with tens.

Partitioning (Augmentation model)

Base 10 equipment:

Encourage the children to begin counting from the first set of ones and tens, avoiding counting from 1. Beginning with the ones in preparation for formal columnar method.



Number line:

At this stage, children can begin to use an informal method to support, record and explain their method. (optional)

30 + 4 + 20 + 3

7

50

57

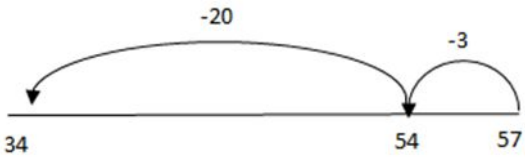
Take away (Separation model)

57 – 23 = 34

Base 10 equipment:

Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.

Number Line:



At this stage, children can began to use an informal method to support, record and explain their method (optional)

(50 + 7) - (20 + 3)

30 4

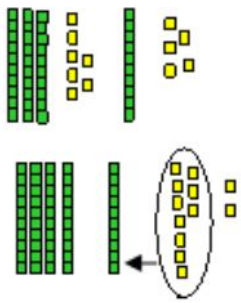
34

Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the ones are greater than 10? Introduce the term 'exchange'. Using the Base 10 equipment, children exchange ten ones for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Base 10 equipment:

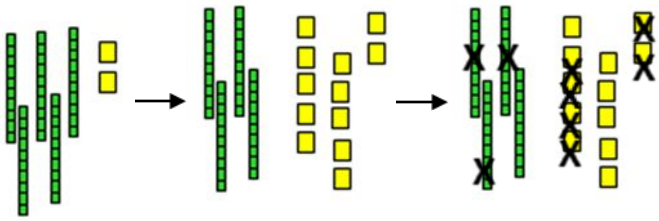
37 + 15 = 52



Discuss counting on from the larger number irrespective of the order of the calculation.

Base 10 equipment:

52 – 37 = 15

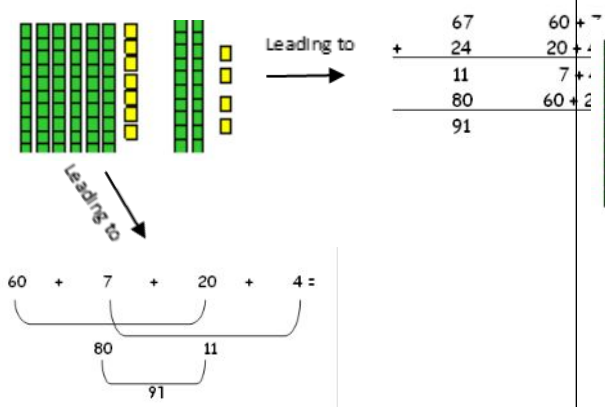


Expanded Vertical Method (optional)

Children are then introduced to the expanded vertical method to ensure that they make the link between using Base 10 equipment, partitioning and recording using this expanded vertical method.

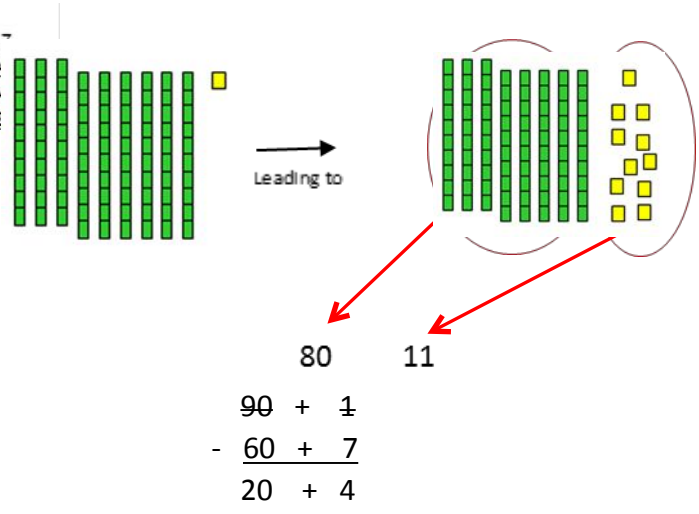
Base 10 equipment:

67 + 24 = 91

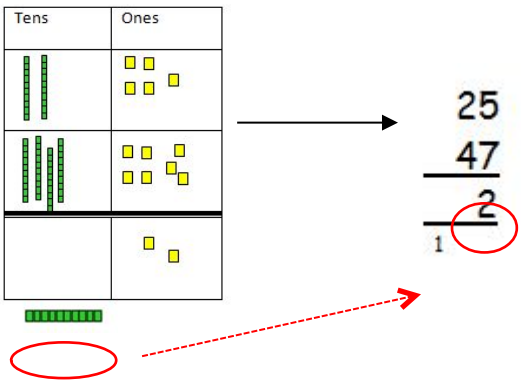


Base 10 equipment:

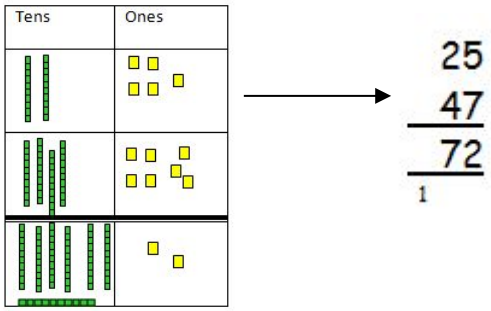
91 – 67 = 24

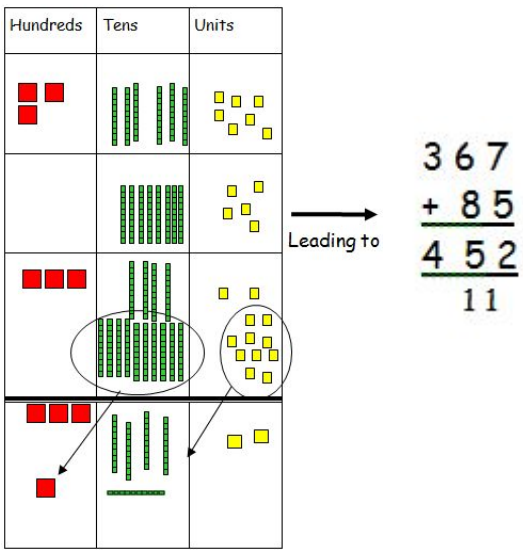


Compact method

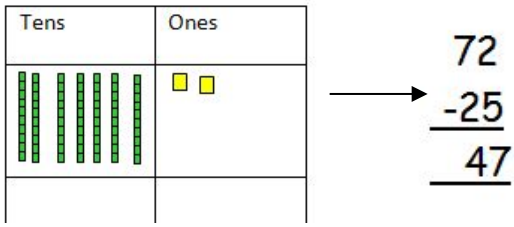


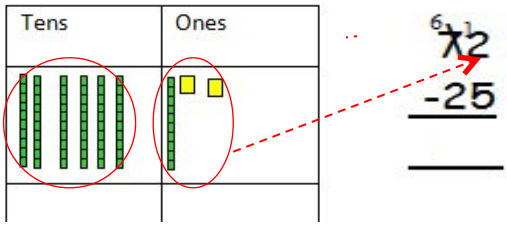
Leading to

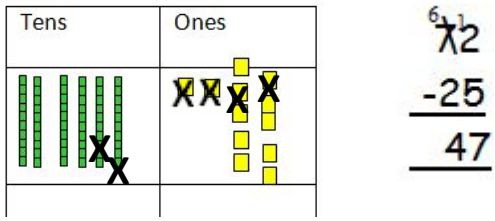




Compact decomposition







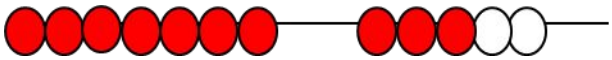
Vertical acceleration

By returning to earlier manipulative experiences children are supported to make links across mathematics, encouraging 'If I know this…then I also know…' thinking.

Decimals

Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.

Bead strings:



Each bead represents 0.1, each different block of colour equal to 1.0

Base 10 equipment:

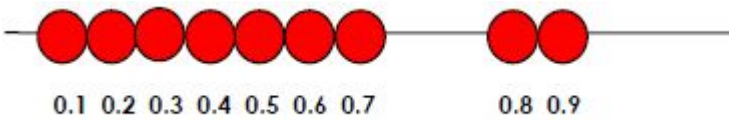
0.1 1.0 10.0

Addition of decimals

Aggregation model of addition

Counting both sets – starting at zero.

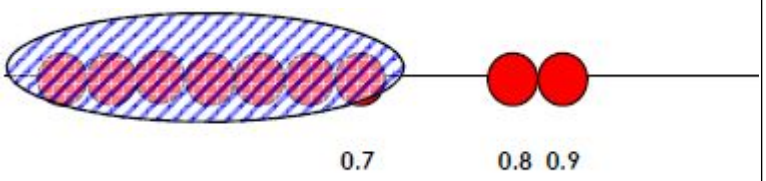
0.7 + 0.2 = 0.9



Augmentation model of addition

Starting from the first set total, count on to the end of the second set.

0.7 + 0.2 = 0.9



Bridging through 1.0

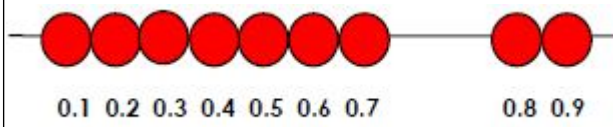
Encouraging connections with number bonds.

0.7 + 0.5 = 1.2

Subtraction of decimals

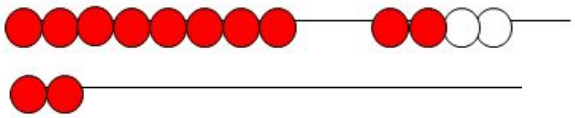
Take away model

0.9 – 0.2 = 0.7



Finding the difference (or comparison model):

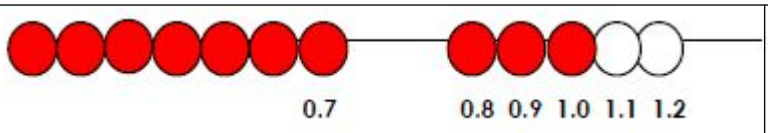
0.8 – 0.2 =



Bridging through 1.0

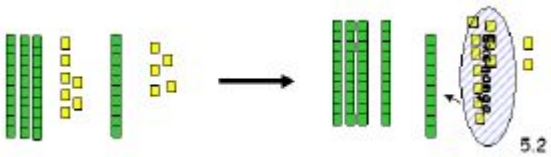
Encourage efficient partitioning.

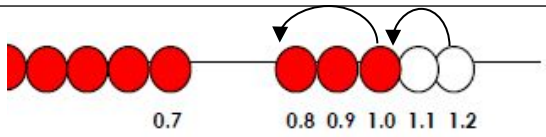
1.2 – 0.5 = 1.2 – 0.2 – 0.3 = 0.7



Partitioning

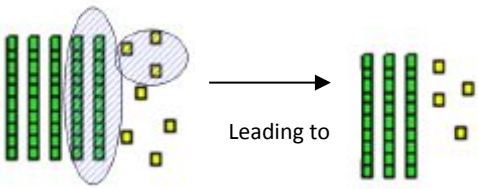
3.7 + 1.5 = 5.2





Partitioning

5.7 – 2.3 = 3.4



Gradation of difficulty- addition:

1. No exchange
2. Extra digit in the answer
3. Exchanging ones to tens
4. Exchanging tens to hundreds
5. Exchanging ones to tens and tens to hundreds
6. More than two numbers in calculation
7. As 6 but with different number of digits
8. Decimals up to 2 decimal places (same number of decimal places)
9. Add two or more decimals with a range of decimal places

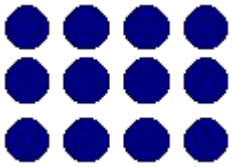
Gradation of difficulty- subtraction:

1. No exchange
2. Fewer digits in the answer
3. Exchanging tens for ones
4. Exchanging hundreds for tens
5. Exchanging hundreds to tens and tens to ones
6. As 5 but with different number of digits
7. Decimals up to 2 decimal places (same number of decimal places)
8. Subtract two or more decimals with a range of decimal places

Progression in Multiplication and Division

Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			



The following array, consisting of four columns and three rows, could be used to represent the number sentences: -

3 x 4 = 12,

4 x 3 =12,

3 + 3 + 3 + 3 = 12,

4 + 4 + 4 =12.

And it is also a model for division

12 ÷ 4 = 3

12 ÷ 3 = 4

12 – 4 – 4 – 4 = 0

12 – 3 – 3 – 3 – 3 = 0

Multiplication	Division
Early experiences Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.	Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.

Repeated addition (repeated aggregation)

3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3

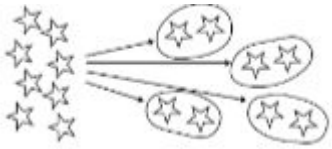
Children learn that repeated addition can be shown on a number line.

Children learn that repeated addition can be shown on a bead string.

Children also learn to partition totals into equal trains using Cuisenaire Rods

Sharing equally

6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.



Grouping or repeated subtraction

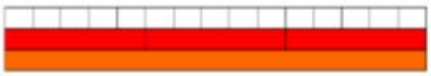
There are 6 sweets. How many people can have 2 sweets each?

Scaling

This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles.

This can be written as:

1 + 1 + 1 = 3 scaled up by 5 5 + 5 + 5 = 15

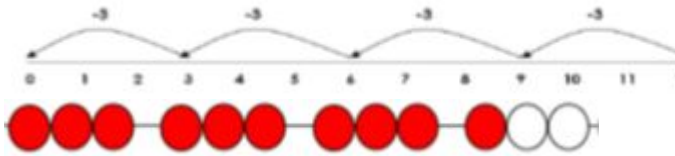


For example, find a ribbon that is 4 times as long as the blue ribbon.

We should also be aware that if we multiply by a number less than 1, this would correspond to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to 3 x 0.5 = 1.5.

Repeated subtraction using a bead string or numbe line

12 ÷ 3 = 4

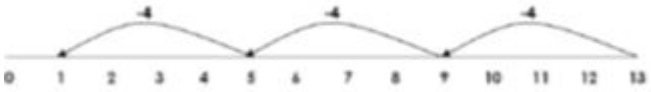
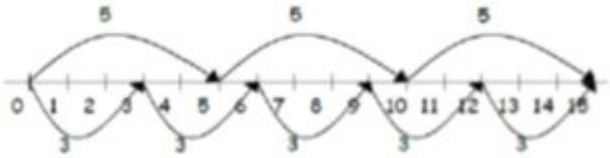
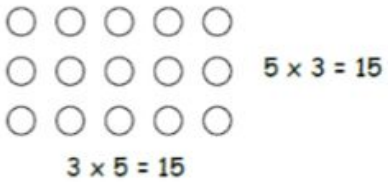
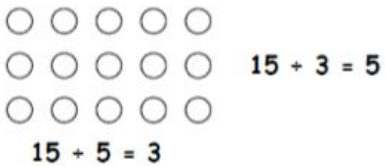
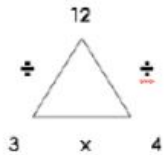
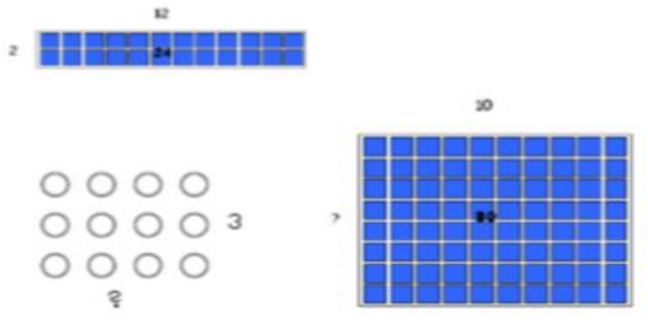


The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as 'how many 3s make 12?'

Cuisenaire Rods also help children to interpret division calculations.

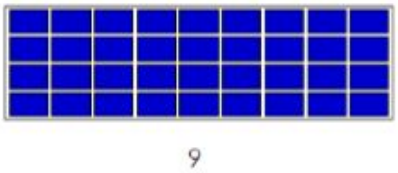
Grouping involving remainders

Children move onto calculations involving remainders.

	13 ÷ 4 = 3 r1  Or using a bead string see above.
Commutativity Children learn that 3 x 5 has the same total as 5 x 3. This can also be shown on the number line. 3 x 5 = 15 5 x 3 = 15 	Children learn that division is **not** commutative and link this to subtraction.
Arrays Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number.  5 x 3 = 15 3 x 5 = 15	Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to **finding fractions** of discrete quantities.  15 ÷ 3 = 5 15 ÷ 5 = 3
Inverse operations Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts. 3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3  Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations. x 5 = 20 3 x Δ = 18 O x = 32 24 ÷ 2 = 15 ÷ O = 3 Δ ÷ 10 = 8	This can also be supported using arrays: e.g. 3 X ? = 12 
Partitioning for multiplication Arrays are also useful to help children visualise how	**Partitioning for division** The array is also a flexible model for division of

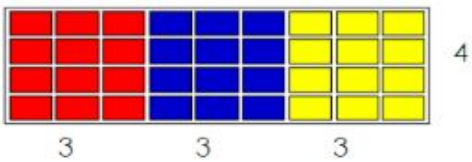
to partition larger numbers into more useful representation.

9 x 4 = 36



Children should be encouraged to be flexible with how they use number and can be encouraged to break the array into more manageable chunks.

9 x 4 =

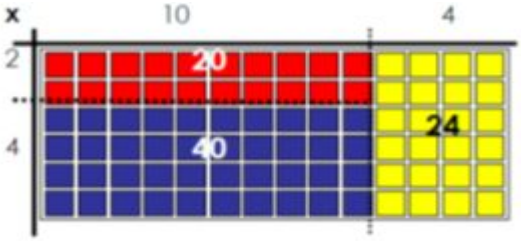


Which could also be seen as

9 x 4 = (3 x 4) + (3 x 4) + (3 x 4) = 12 + 12 + 12 = 36

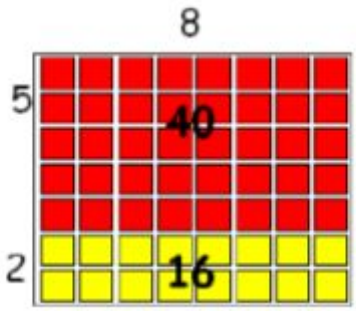
Or 3 x (3 x 4) = 36

And so 6 x 14 = (2 x 10) + (4 x 10) + (4 x 6) = 20 + 40 + 24 = 84



larger numbers

56 ÷ 8 = 7



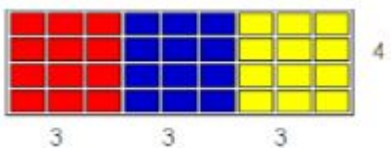
Children could break this down into more manageable arrays, as well as using their understanding of the inverse relationship between division and multiplication.

56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8) = 5 + 2 = 7

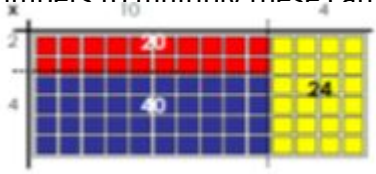
To be successful in calculation learners must have plenty of experiences of being flexible with partitioning, as this is the basis of distributive and associative law.

Associative law (multiplication only) :-

E.g. 3 x (3 x 4) = 36



The principle that if there are three numbers to multiply these can be multiplied in any order.



Distributive law (multiplication):-

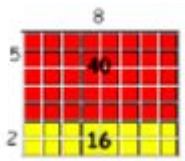
E.g. 6 x 14 = (2 x 10) + (4 x 10) + (4 x 6) = 20 + 40 + 24 = 84

This law allows you to distribute a multiplication across an addition or subtra

Distributive law (division):-

E.g. 56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8) = 5 + 2 = 7

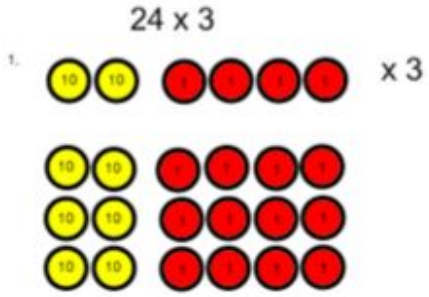
This law allows you to distribute a division across an addition or subtraction.



Arrays leading into the grid method

Children continue to use arrays and partitioning, where appropriate, to prepare them for the grid method of multiplication.

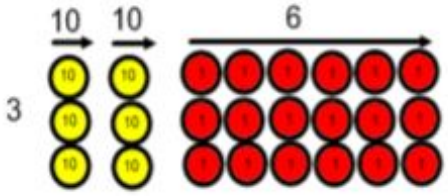
Arrays can be represented as 'grids' in a shorthand version and by using place value counters to show multiples of ten, hundred etc.



Arrays leading into chunking and then long and short division

Children continue to use arrays and partitioning where appropriate, to prepare them for the 'chunking' and short method of division. Arrays are represented as 'grids' as a shorthand version.

e.g. 78 ÷ 3 =

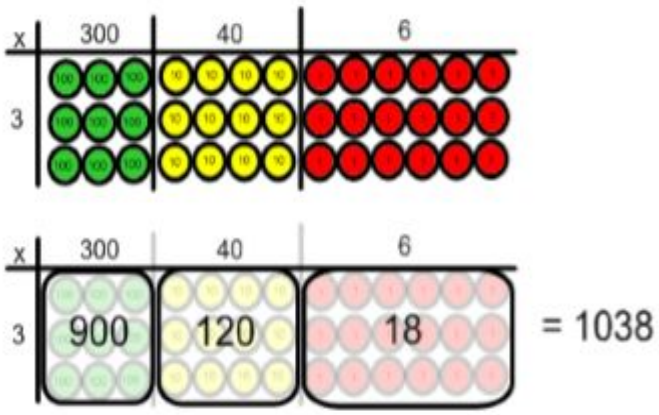


78 ÷ 3 = (30 ÷ 3) + (30 ÷ 3) + (18 ÷ 3) =

10 + 10 + 6 = 26

Grid method

This written strategy is introduced for the multiplication of TO x O to begin with. It may require column addition methods to calculate the total.



The vertical method- 'chunking' leading to long division

See above for example of how this can be modelled as an array using place value counters.

78 ÷ 3 =

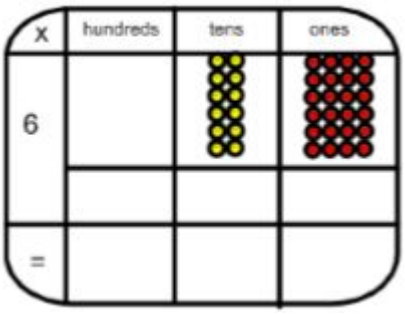
	78	
-	30	(10 x 3)
	48	
-	30	(10 x 3)
	18	
-	18	(6 x 3)
	0	

So 78 ÷ 3 = 10 + 10 + 6 =26

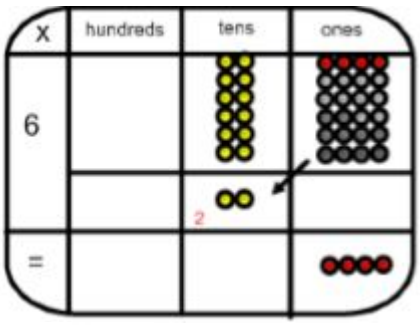
Short multiplication — multiplying by a single digit

The array using place value counters becomes the basis for understanding short multiplication first without exchange before moving onto exchanging

24 x 6



$$\begin{array}{r} 24 \\ \times\ 6 \\ \hline \end{array}$$

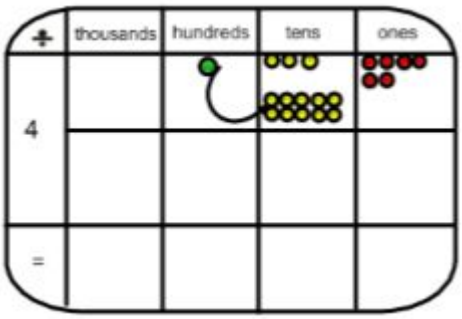


$$\begin{array}{r} 24 \\ \times\ 6 \\ \hline 4 \\ \hline 2 \end{array}$$

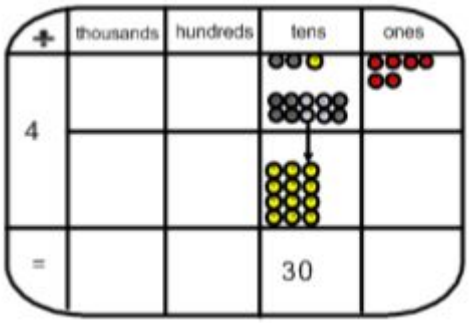
Short division — dividing by a single digit

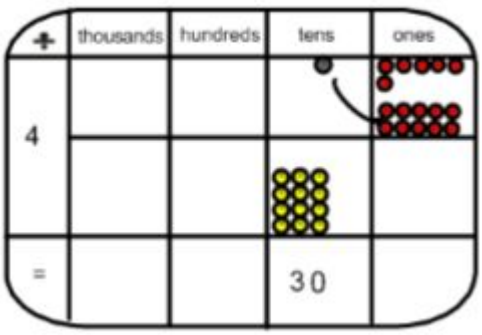
Whereas we can begin to group counters into an array to show short division working

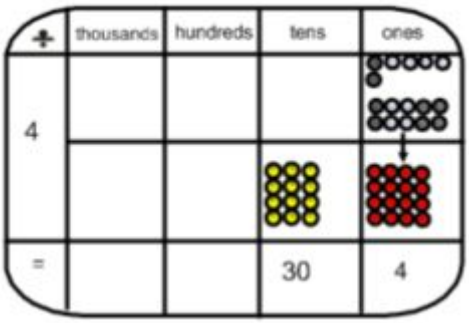
136 ÷ 4



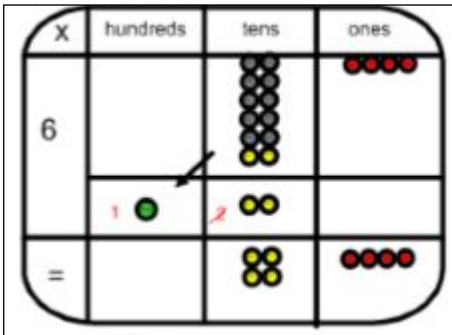
$$4\overline{)136}\quad 3$$





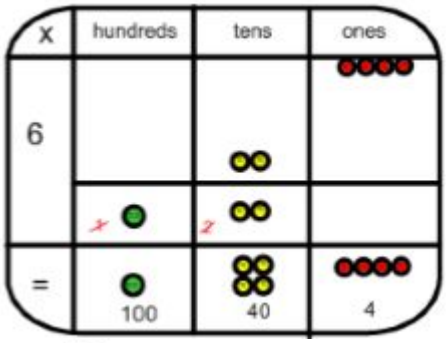


$$4\overline{)136}\quad 34$$



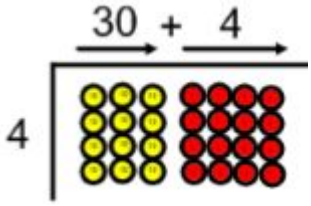
24

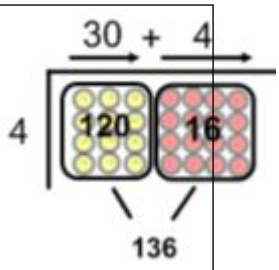
$$\begin{array}{r} \times\ 6 \\ \hline 44 \\ \hline \end{array}$$



24

$$\begin{array}{r} \times\ 6 \\ \hline 144 \\ \hline \end{array}$$





Gradation of difficulty (short multiplication)	**Gradation of difficulty (short division)**
1. TO x O no exchange 2. TO x O extra digit in the answer 3. TO x O with exchange of ones into tens 4. HTO x O no exchange 5. HTO x O with exchange of ones into tens	1. TO ÷ O no exchange no remainder 2. TO ÷ O no exchange with remainder 3. TO ÷ O with exchange no remainder 4. TO ÷ O with exchange, with remainder 5. Zero in the quotient e.g. 816 ÷ 4 = **204** 6. As 1-5 HTO ÷ O

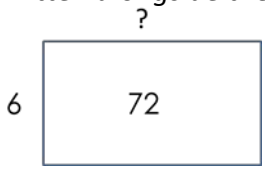
6. HTO x O with exchange of tens into hundreds 7. HTO x O with exchange of ones into tens and tens into hundreds 8. As 4-7 but with greater number digits x O 9. O.t x O no exchange 10. O.t with exchange of tenths to ones 11. As 9 - 10 but with greater number of digits which may include a range of decimal places x O	7. As 1-5 greater number of digits ÷ O 8. As 1-5 with a decimal dividend e.g. 7.5 ÷ 5 or 0.12 ÷ 3 9. Where the divisor is a two digit number See below for gradation of difficulty with remainders
	<u>Dealing with remainders</u> Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly. e.g.: · I have 62p. How many 8p sweets can I buy? · Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed? **<u>Gradation of difficulty for expressing remainders</u>** 1. Whole number remainder 2. Remainder expressed as a fraction of the divisor 3. Remainder expressed as a simplified fraction 4. Remainder expressed as a decimal

<u>Long multiplication—multiplying by more than one digit</u> Children will refer back to grid method by using place value counters or Base 10 equipment with no exchange and using	**<u>Long division —dividing by more than one digit</u>** Children should be reminded about partitioning numbers into multiples of 10, 100 etc. before recording as either:-

synchronised modelling of written recording as a long multiplication model before moving to TO x TO etc.	1. Chunking model of long division using Base 10 equipment 2. Sharing model of long division using place value counters See the following pages for exemplification of these methods.

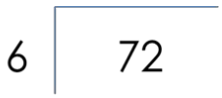
Chunking model of long division using Base 10 equipment

This model links strongly to the array representation; so for the calculation 72 ÷ 6 = ? - one side of the array is unknown and by arranging the Base 10 equipment to make the array we can discover this unknown. The written method should be written alongside the equipment so that children make links.

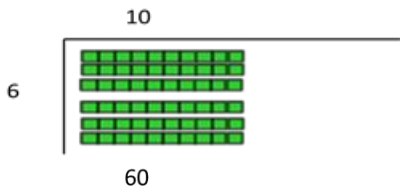


Begin with divisors that are between 5 and 9

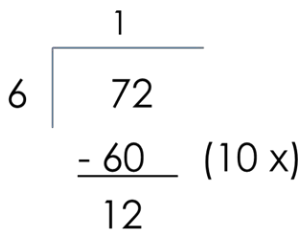
72 ÷ 6 = 12



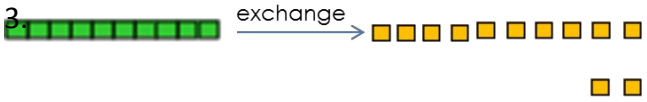
1. Make a rectangle where one side is 6 (the number dividing by) – grouping 6 tens



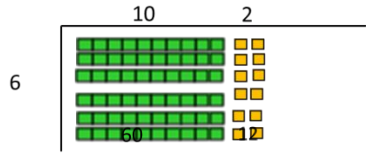
After grouping 6 lots of 10 (60) we have 12 left over



2. Exchange the remaining ten for ten ones

3. 

4. Complete the rectangle by grouping the remaining ones into groups of 6

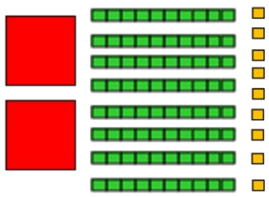


$$\begin{array}{r|l} & 12 \\ \hline 6 & 72 \\ & -\ 60 \quad (10\ x) \\ & 12 \\ & -\ 12 \quad (2\ x) \\ & 0 \end{array}$$

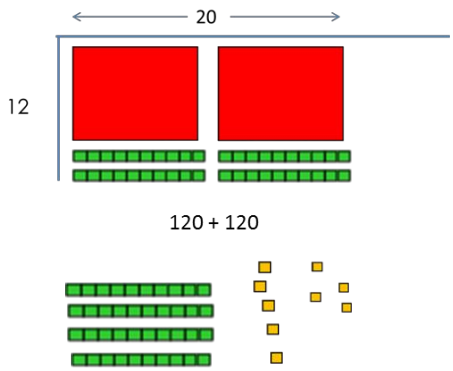
Move onto working with divisors between 11 and 19

Children may benefit from practise to make multiples of tens using the hundreds and tens and tens and ones.

289 ÷ 12



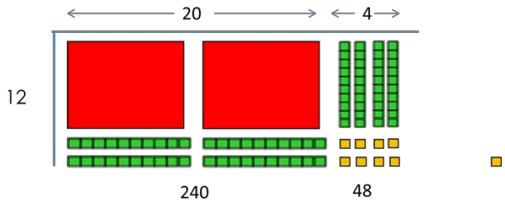
1. Make a rectangle where one side is 12 (the number dividing by) using hundreds and tens



With 49 remaining

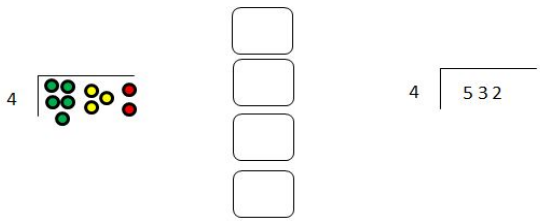
$$\begin{array}{r|l} 12 & 289 \end{array}$$

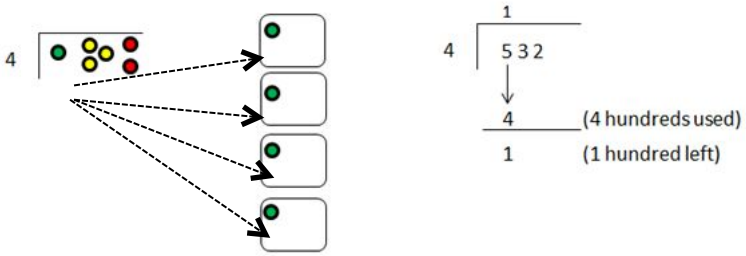
$$\begin{array}{r|l} & 2 \\ \hline 12 & 289 \\ - & 240 \quad (20\ x\ ) \\ & 49 \end{array}$$

2. Make groups of 12 using tens and ones  No more groups of 12 can be made and 1 remains	24 r1 12 | 289 - 240 (20 x) 49 - 48 (4 x) 1

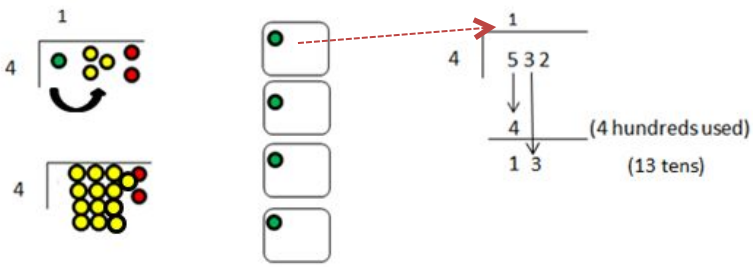
Sharing model of long division using place value counters

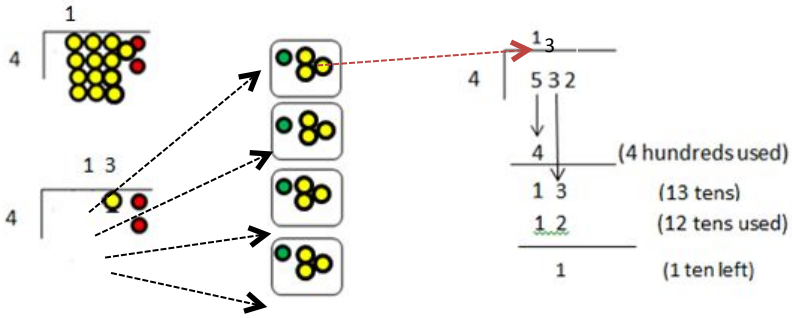
Starting with the most significant digit, share the hundreds. The writing in brackets is for





Moving to tens – exchanging hundreds for tens means that we now have a total of 13 tens





Moving to ones, exchange tens to ones means that we now have a total of 12 ones counters (hence the arrow)